Gravity as field in the Planck limit.

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We show that Geroch decomposition leads us to define Maxwell-like representation of gravity in $(3+1)$ metrics decomposition. From such decomposition we obtain scalar field that may be assigned with gravitational interaction. We use this field to propose new approach to quantization of gravity that results in proper quanta values.

1 Introduction

There are known scalar theories of gravity on flat spacetime [1] explaining gravity as interaction between bodies. In last 50 years many of them start to be considered as alternative to classical general relativity. In all of this approaches authors try to extend gravity to the Planck limit. For this efforts the main problem in our opinion is effective quantization of this theories. In our approach we propose first to extend classical general relativity to Planck limit on (3+1) manifold and next quantize it in classical and covariant way.

In first part we show, that Geroch decomposition for spherically symmetric case, opens new way to understand the curved spacetime as the effect of local interaction of flat spacetime. In this perspective gravity is described by effective scalar field $\Phi = 1/\gamma_r$, which can be interpreted - in naive approach - as gravitational time dilatation acting on flat spacetime.

In second part of this article we show, that Φ may also serve us to describe classical electromagnetic field. We show that in infinity limit we are able to reconstruct electromagnetic equations in classic and covariant form.

In third part of the article we show how we may quantize the field Φ to obtain proper rest mass, photon energy and Coulomb-like potential for elementary charges.

We hope that this approach may bring important implications for our understanding of spacetime in zero limit and may shed new light on quantum gravity theories and opens new areas for research and generalizations.

2 Local surrounding in Minkowski spacetime as scalar field.

2.1 Killing vector fields

At the beginning we define, Einstein summation convention. Commas denote partial derivatives:

$$\varphi_{,\mu} = \frac{\partial \varphi}{\partial x^{\mu}} \tag{1}$$

Semicolons denote covariance derivative:

$$\nabla_{\mu} X^{\alpha} = X^{\alpha}_{;\mu} = X^{\alpha}_{,\mu} + \Gamma^{\alpha}_{\mu\nu} X^{\nu} \tag{2}$$

where Γ 's are the connection coefficients. We choose metric signature $(-,+,+,+)$. The geodesic equation:

$$0 = \frac{dx^{\mu}}{d\tau}\left[\left(\frac{dx^{\alpha}}{d\tau}\right)_{,\mu} + \Gamma^{\alpha}_{\mu\nu}\frac{dx^{\nu}}{d\tau}\right] \tag{3}$$

states that the covariance derivative of the particle four-velocity:

$$u^{\mu} = \frac{dx^{\mu}}{d\tau} \tag{4}$$

along itself vanishes:

$$0 = \nabla_{\mu} u^{\alpha} = u^{\mu} u^{\alpha}_{;\mu} = \frac{dx^{\alpha}}{d\tau^2} + \Gamma^{\alpha}_{\mu\nu}\frac{dx^{\mu}}{d\tau}\frac{dx^{\nu}}{d\tau} \tag{5}$$

Here τ is affine parameter, which is proper time for timelike geodesics. On a spacetime, a Killing vector field [2]

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generates an isometry of spacetime. Generally, this requires solving the equation

$$\xi_{\mu;\nu} + \xi_{\nu;\mu} = 0 \tag{6}$$

but in the case where we have a coordinate chart in which the metric coefficients are independent of a coordinate, then the vector field of that coordinate is automatically a Killing field.

For a geodesic, it defines a constant of motion, since

$$\nabla_\mu \left(u^\mu \cdot \xi\right) = u^\nu \left(u^\mu \xi_\mu\right)_{;\nu} = 0 \tag{7}$$

$$u^\nu u^\mu_{;\nu} \xi_\mu + u^\nu u^\mu \xi_{\mu;\nu} = 0 \tag{8}$$

the first term being zero because of the geodesic equation and the second term because of anti-symmetry of $\xi_{\mu;\nu}$

To introduce Killing fields we need to choose metric for which we can find at lest one vector ξ_μ. For the simplicity let us introduce Schwarzschild spacetime in the easy form for interpretation:

$$ds^2 = -\left(1 - \frac{r_s}{r}\right) dt^2 + \frac{dr^2}{\left(1 - \frac{r_s}{r}\right)} + r^2 \left(d\theta^2 + \sin^2\theta \, d\varphi^2\right) \tag{9}$$

We see that the metric coefficient $\left(1 - \frac{r_s}{r}\right)$ is time t and φ independent, so that ∂_t and ∂_φ are Killing fields.

In Schwarzschild coordinates, a particle worldline has the four-velocity

$$u^\mu = \left[\frac{dt}{d\tau}; \frac{dr}{d\tau}; \frac{d\theta}{d\tau}; \frac{d\varphi}{d\tau}\right] \tag{10}$$

so for t:

$$\xi^\mu = [1;0;0;0] \tag{11}$$

we have the dot product

$$-e_0 = g_{\mu\nu} \xi^\mu u^\nu = -\left(1 - \frac{r_s}{r}\right) \frac{dt}{d\tau} \tag{12}$$

and similarly for φ:

$$\xi^\mu = [0;0;0;1] \tag{13}$$

we have

$$l = e_\varphi = g_{\mu\nu} \xi^\mu u^\nu = r^2 \left(\sin^2\theta\right) \frac{d\varphi}{d\tau} \tag{14}$$

The physical meaning is that e_0 and l are conserved specific (per-mass) energy and azimuthal angular momentum of the particle at infinity, respectively.

We should also note, that this is relativistic energy, so for free-fall from rest at infinity (with escape velocity) we have $e_0 = 1$

2.2 Geroch decomposition

Let us apply Geroch decomposition [3] for chosen Schwarzschild spacetime in spherically symmetric case and discuss implications of that operation.

Schwarzschild metric is an asymptotically flat spacetime with a timelike Killing vector field $\mu = [1;0;0;0]$ with norm-squared

$$\lambda = -\xi^\mu \xi_\mu \tag{15}$$

and twist

$$\omega_\mu = \epsilon_{\mu\nu\rho\delta} \xi^\nu \nabla^\rho \xi^\delta \tag{16}$$

using the tensor

$$\gamma_{\mu\nu} = \lambda g_{\mu\nu} + \xi_\mu \xi_\nu \tag{17}$$

the spacetime metric takes the form:

$$ds^2 = -\lambda \left(dt - \omega_i dx^i\right)^2 + \frac{\gamma_{ij}}{\lambda} dx^i dx^j \tag{18}$$

3 Maxwell-like representation of gravity in $(3+1)$ metrics decomposition

3.1 Geroch decomposition representation

Let us rewrite metric (18) in more straightforward form:

$$ds^2 = -\lambda dt^2 + 2\lambda \omega_i dt dx^i - (\lambda \omega_i \omega_j + \frac{\gamma_{ij}}{\lambda}) dx^i dx^j \tag{19}$$

This is the metric in form of:

$$ds^2 = g_{00} dt^2 + 2 g_{0i} dt dx^i + g_{\alpha\beta} dx^i dx^j \tag{20}$$

For convention let us introduce:

$$h \equiv g_{00} = -\lambda \tag{21}$$

$$g_\alpha \equiv -\frac{g_{0\alpha}}{g_{00}} = -2\lambda\omega_i \tag{22}$$

$$\Upsilon_{\alpha\beta} \equiv -g_{\alpha\beta} + h g_\alpha g_\beta = -(\frac{\gamma_{ij}}{\lambda} + \lambda \omega_i \omega_j) \tag{23}$$

For this definitions we can introduce Maxwell-like equations in (3+1) form:

$$F_{[ik;l]} = 0 \tag{24}$$

In more classical way:

$$\nabla \cdot \mathbf{B} = 0 \tag{25}$$

$$\nabla \times \mathbf{E} = -\frac{1}{\Upsilon}\frac{\partial}{\partial t}\left(\sqrt{\Upsilon}\mathbf{B}\right) \tag{26}$$

For better understanding let us write:

$$\frac{1}{\sqrt{-g}}\partial_a\left(\sqrt{-g}F^{ab}\right) = 0 \tag{27}$$

then Maxwell-like equations can be rewritten as:

$$\nabla \cdot \mathbf{D} = 0 \tag{28}$$

$$\nabla \times \mathbf{H} = -\frac{1}{\Upsilon}\frac{\partial}{\partial t}\left(\sqrt{\Upsilon}\mathbf{D}\right) \tag{29}$$

where:

$$\mathbf{D} = \frac{\mathbf{E}}{\sqrt{h}} + \mathbf{H} \times \mathbf{g} \tag{30}$$

$$\mathbf{B} = \frac{\mathbf{H}}{\sqrt{h}} + \mathbf{g} \times \mathbf{E} \tag{31}$$

Schwarzschild metric is static and components of it are time independent so we can rewrite Maxwell-like equations as:

$$\nabla \cdot \mathbf{B} = 0 \tag{32}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{33}$$

$$\nabla \cdot \left(\frac{\mathbf{E}}{\sqrt{h}}\right) = 0 \tag{34}$$

$$\nabla \times \sqrt{\mathbf{h}}\mathbf{B} = \frac{1}{\sqrt{h}}\frac{\partial \mathbf{E}}{\partial t} \tag{35}$$

$$\tag{36}$$

Above equations acts like Maxwell equations, where the speed of wave propagation is

$$V_w = \sqrt{h} = c\sqrt{1 - \frac{r_s}{r}} \tag{37}$$

It is simple in this moment to show that

$$\lim_{r\to\infty} V_w = c \tag{38}$$

3.2 Propagating disturbances of spacetime isometry

Killing vector X^b by definition [2] satisfies:

$$g^{bc}X_{c;ab} - R_{ab}X^b = 0 \text{ [13, p. 443; C.3.9]} \tag{39}$$

$$X_{a;bc} = R_{abcd}X^d \text{ [13, p. 443; C.3.6]} \tag{40}$$

$$X^{a;b}_{;b} + R^a_c X^c = 0 \tag{41}$$

Thus for introduced Killing field we have:

$$R^{\alpha}_{\beta\gamma\sigma}\xi^{\sigma} = \xi^{\sigma}_{;\beta;\gamma} \tag{42}$$

Therefore:

$$\xi^{;\beta}_{\alpha;\beta} = -R_{\alpha\beta}\xi^{\beta} \tag{43}$$

so the timelike Killing field is intimately connected to spacetime curvature. Defining a convenient F and using the defining property of Killing fields

$$\xi_{\alpha;\beta} + \xi_{\beta;\alpha} = 0 \tag{44}$$

we obtain

$$F_{\alpha\beta} = \xi_{[\beta;\alpha]} = \frac{1}{2}\left(\xi_{\beta;\alpha} - \xi_{\alpha;\beta}\right) = -\xi_{\alpha;\beta} \tag{45}$$

therefore

$$F^{;\beta}_{\alpha\beta} = R_{\alpha\beta}\xi^{\beta} \tag{46}$$

This looks like the standard electromagnetic field tensor

$$F^{;\beta}_{\alpha\beta} = A_{\beta;\alpha} - A_{\alpha;\beta} \tag{47}$$

which couples to four-current J through Maxwell's equation

$$F^{\alpha\beta}_{;\beta} = 4\pi J^{\alpha} \tag{48}$$

In vacuum, the Ricci tensor vanishes, and the Killing field ξ seems to act like the electromagnetic four-potential A that acts for electromagnetism in source-free regions, in the Lorentz gauge

$$A^{\alpha}_{;\alpha} = 0 \tag{49}$$

which automatically satisfies all Maxwell's equations. Physical meaning of the tensor F should be explained as propagating spacetime anisometry.

3.3 Maxwell-like wave equations

3.3.1 Gravity as wave equation in Minkowski spacetime

Let us follow [4] and consider regular, flat Minkowski spacetime. We define scalar potentials:

$$\Phi = \frac{1}{\gamma_r} = \sqrt{1 - \frac{r_s}{r}} \tag{50}$$

$$\Theta = r \cdot \beta_r = r\sqrt{\frac{r_s}{r}} \tag{51}$$

We define following vector fields (where $\hat{e}$ are directional versors):

$$\mathbf{T} = c\Phi \cdot \hat{e}_y \tag{52}$$

$$\mathbf{A} = -\nabla c\Phi \times \hat{e}_y = -\nabla \times \mathbf{T} \tag{53}$$

$$\mathbf{U} = \nabla c\Theta \times \hat{e}_x \tag{54}$$

$$\Omega = \nabla \times \mathbf{U} = \frac{d\hat{e}_y}{dt} \tag{55}$$

Using relations between above fields we obtain:

$$\frac{\gamma_r}{c} \cdot \frac{d\mathbf{T}}{dt} = \frac{\gamma_r}{c}\frac{c}{\gamma_r} \cdot \frac{d\hat{e}_y}{dt} = \Omega \tag{56}$$

$$\nabla \times \mathbf{A} = \frac{\gamma_r}{c} \cdot \frac{d\Omega}{dt} \tag{57}$$

After simple transformations we derive d'Alambertian:

$$\frac{1}{c^2} \cdot \frac{\partial^2 \mathbf{A}}{\partial \tau^2} - \nabla^2 \mathbf{A} = 0 \tag{58}$$

or with the same meaning:

$$\frac{\gamma_r^2}{c^2} \cdot \frac{\partial^2 \mathbf{A}}{\partial t^2} - \nabla^2 \mathbf{A} = 0 \tag{59}$$

Let us notice, that in our description time flow (t) axis is orthogonal to spatial axis but acts as rotating field. Therefore we may still consider both as Minkowski spacetime with orthogonal space and time.

With respect to some factor above d'Alambertian should be able to work as electromagnetic wave description.

3.3.2 Interpretation of Maxwell-like picture of metric

We have constructed a timelike Killing field what means that we have a stationary spacetime.

For now, let us assume that the Killing field is irrotational:

$$\omega_i = 0 \tag{60}$$

Then spacetime is static, and the Killing observers (four-velocities parallel to the timelike Killing field) are also static, since:

$$dx^\mu = 0 \tag{61}$$

If we normalize the four-velocity properly:

$$u^\mu = \lambda^{-1/2}\xi^\mu \tag{62}$$

Thus

$$\lambda^{-1/2}_{Schwarzschild} = \frac{1}{\sqrt{1-\frac{r_s}{r}}} = \gamma_r \tag{63}$$

We see that this quantity is the inverse-norm of our timelike Killing field. Without the inversion, the quantity $\lambda^{1/2}$ is the gravitational redshift.

The four-acceleration of the Killing observers given by a covariant derivative in our static case simplifies to just:

$$a^\mu = U^{;\mu} = g^{\mu\nu}\left(\log \lambda^{1/2}\right)_{,\nu} \tag{64}$$

In the Schwarzschild case, everything but the r-component vanishes, and we can put it in corresponding orthonormal basis rather the coordinate vector:

$$\partial_r \hat{e}_r = \partial_r\left(\sqrt{1-\frac{r_s}{r}}\partial_r\right) \tag{65}$$

wich yealds to:

$$a^r = \left(1-\frac{r_s}{r}\right)\frac{1}{2}\frac{r_s}{r^2}\frac{1}{1-\frac{r_s}{r}} = \frac{r_s}{2r^2} \tag{66}$$

Thus

$$a = \frac{r_s}{2r^2}\frac{1}{\sqrt{1-\frac{r_s}{r}}}\hat{e}_r = g_r\hat{e}_r \tag{67}$$

showing the correct gravitational acceleration: the proper acceleration of the Killing observers is g_r into the outwardly radial direction.

Of course this is only conceptual interpretation of our (3+1) decomposition of Schwarzschild metric. In this moment we have to stress that depending on coordinate system we can get different interpretation of gravity in pseudo classical picture.

3.4 Action of the scalar field

Taking t=0 hypersurface, the symmetry of the spacetime under time inversion means that the extrinsic curvature is zero, in which case the Gauss-Codazzi equations simplify to:

$$R^a_{bcd} =^h R^a_{bcd} \tag{68}$$

$$R^0_{bcd} = 0 \tag{69}$$

$$R^0_{i0j} = -^h U_{;i;j} - \left(^h U_{;i}\right)\left(^h U_{;j}\right) \tag{70}$$

where the superscript h denotes that the quantity belongs to spatial hyperslice and should be calculated using the spatial metric h alone.

Contracting all the way down to the Ricci scalar:

$$R =^h R - 2\left[^h U^{;i}_{;i} + \left(^h U^{;i}\right)\left(^h U_{;i}\right)\right] \tag{71}$$

The first term in the bracket is a Laplacian

$$2U = \log\lambda \tag{72}$$

and the metric determinant is

$$g = -\lambda h = -\gamma \tag{73}$$

so in terms of the metric:

$$\gamma_{ij}\left(\frac{1}{\lambda}\right)h_{ij} \tag{74}$$

the representation of the Einstein-Hilbert action is

$$S\,\alpha \int R\sqrt{-g} \tag{75}$$

$$S = \int \left(^\gamma R -^\gamma \nabla^2 (\log\lambda) - \frac{1}{2}\frac{\gamma_{ij} d\lambda^i d\lambda^j}{\lambda^2}\right)\sqrt{\gamma} \tag{76}$$

We have obtained a proper result.

We have just shown that in the Geroch decomposition of Schwarzschild metric we can consider gravity as medium that change the speed of light in Maxwell-like picture of gravity, but, of course, in local coordinate system speed of light is still constant and equal to c.

This consideration leads us to the conclusion that after Geroch decomposition we can reconsider gravity not only as a spacetime curvature but also as some field $\Phi = \sqrt{h} = 1/\gamma_r$. Of course with this picture of gravity we can only get in (3+1) Geroch decomposition. But it means, that in Geroch picture we get that gravity is Maxwell-like field and equation of motion of this field is dependent on what coordinate system we choose.

We have to be aware of this property of the gravity. If we try to interpret it as some field in 4 or (3+1) dimensions we first have to decide what is the metric and in what coordinate system we consider this metric. In naive picture we can say that:

- metric choose is equivalent to choose of field (conservation laws),

- choose of coordinate system is equivalent to choose of equation of motion (dynamic equations).

4 Quantum picture of the scalar field Φ

As it was shown, the interpretation of our problem will depend not only on metric we choose but also on coordinate system. This remark is our first condition if we would like to quantize classical field that we present in section 3.

The choose of action of the field presented in section 3.4 is not sufficient. To do quantization of gravity we have to introduce Hamiltonian in specific coordinate system for which it will be obvious how in classical way we can do the process of quantization.

Still it is not obvious how to choose Hamiltonian in a way that will give us equation of motion for operators and wave functions that we could easy interpret using the Copenhagen interpretation of wave function.

In first approach we just use action to define Hamiltonian in form where we have separated time and space in linear form and then quantize it.

In second approach we can write Hamiltonian in covariant way and choose condition for metric that will give us confidence that our equation of motion will be well defined.

Now, we try to show how those two different approaches works and what result they give.

4.1 Classical quantization - first approach

From equation (76) we may deduce that gravitational action of a static spacetime is equivalent to the action of a 3-dimensional spacelike manifold minimally coupled to a scalar field $\Phi = 1/\gamma_r$. Let us define Lagrangian and Hamiltonian for that situation.

As we know by definition:

$$H = \sum_i \dot{x}_i \frac{\partial L}{\partial \dot{q}_i} - L = \sum_i \dot{x}_i p_i - L \tag{77}$$

For some test body with rest mass "m" moving in flat 3 dimentional space we would have:

$$\sum_{i=1}^{3} \dot{x}_i \frac{\partial L}{\partial \dot{q}_i} = v \cdot m v \gamma = mc^2 \beta^2 \gamma \tag{78}$$

where

$$\beta = \frac{v}{c} \tag{79}$$

Let us define zero-dimension (time dimension) to obtain:

$$\dot{x}_0 p_0 = -v_r \cdot m v_r \gamma_r = -mc^2 \beta_r^2 \gamma_r \tag{80}$$

Now, utilizing definition (77) by simple calculations we may define proper Lagrangian an Hamiltonian for ($\mu = 0,1,2,3$) in form of:

$$L = mc^2 \frac{1}{\gamma_r} - mc^2 \frac{1}{\gamma} \tag{81}$$

we obtain Hamiltonian in form of:

$$H = \sum_\mu \dot{x}_\mu \frac{\partial L}{\partial \dot{q}_\mu} - L \tag{82}$$

$$H = mc^2 \beta^2 \gamma - mc^2 \beta_r^2 \gamma_r - mc^2 \frac{1}{\gamma_r} + mc^2 \frac{1}{\gamma} \tag{83}$$

$$H = mc^2 \left(\beta^2 \gamma + \frac{1}{\gamma} \right) - mc^2 \left(\beta_r^2 \gamma_r + \frac{1}{\gamma_r} \right) \tag{84}$$

$$H = mc^2 \gamma - mc^2 \gamma_r \tag{85}$$

To comply with the Newton approximation we will note it in form of:

$$H = mc^2 (\gamma - 1) - V(r) \tag{86}$$

where

$$V(r) = mc^2 (\gamma_r - 1) \tag{87}$$

We may now follow Schrodinger's way, approximating kinetic energy and expressing it with momentum in classical form:

$$H = \frac{p^2}{2m} - V(r) \tag{88}$$

We may also try to reproduce relativistic Dirac equation, however for our analysis Schrodinger approximation appears to be sufficient. In this moment we have well defined, approximated Hamiltonian that we can easily quantize using relation:

$$i\hbar \frac{\partial}{\partial t} \Psi = \hat{H} \Psi \tag{89}$$

Which give:

$$i\hbar \frac{\partial}{\partial t} \Psi(r,t) = \left[\frac{\hbar^2}{2m} \nabla^2 - V(r) \right] \Psi(\mathbf{r},t) \tag{90}$$

where we use by definition:

$$p = -i\hbar \nabla \tag{91}$$

As we can see this is Schwarzschild like quantum equation. In this classical form of equation we can interpret wave function Ψ in classical way and look on square of it as on probability of finding particle.

It is important to show here that using Maclaurin's expansion in the limit we get:

$$\lim_{r\to\infty} V(r) = mc^2 \left(0 - \frac{r_s}{2r} - \frac{3{r_s}^2}{8r^2} + O\left(\left(\frac{1}{r} \right)^3 \right) \right) \tag{92}$$

In this limit of potential we may easy recognize, that in zero approximation we get free moving particle with mass m. In first approximation we get particle that interacts with scalar field of $1/r$ type.

In our approach we have any particle in spacetime. We see that in quantum approach we get that gravity in first approximation acts on particle as scalar field and can create nonzero stable configuration base state.

Mass of our particle can go to zero or to the limit of Planck mass. In this case we get vacuum solution of our quantum equation with virtual particle which effective mass will be higher than zero but in general different then Planck mass.

Therefore it is worth to show, that calculating limits of $V(r)$ for Planck scales we obtain (after approximation with Maclaurin's expansion) valid classical approximation of rest mass, Newton's gravitational potential, photon energy and electrostatic Coulomb-like potential of two elementary charges.

4.1.1 Rest mass approximation and Newton's limit of potential

For $r_s << l_P$ we calculate:

$$\lim_{m \to m_P; r \to l_P} V(r) = \tag{93}$$

$$= m_P c^2 \left(\frac{1}{\sqrt{1 - \frac{r_s}{l_P}}} - 1 \right) \approx m_P \cdot \frac{c^2 r_s}{2 l_P} = \frac{c^4 r_s}{2G} = Mc^2 \tag{94}$$

We have obtained value that may be treated as some rest energy (some rest mass M) related to given Schwarzschild radius.

We obtain Newton's limit of gravitational potential by expression:

$$Mc^2(\gamma_r - 1) \approx M \frac{c^2 r_s}{2r} = G \frac{mM}{r} \tag{95}$$

4.1.2 Photon energy approximation

For $r >> l_P$ we calculate:

$$\lim_{m \to m_P; r_s \to l_P} V(r) = \tag{96}$$

$$= m_P c^2 \left(\frac{1}{\sqrt{1 - \frac{l_P}{r}}} - 1 \right) \approx m_P c^2 \frac{l_P}{2r} = \frac{1}{2} \hbar \omega \tag{97}$$

where

$$\omega = \frac{c}{r} = \frac{2\pi}{T} \tag{98}$$

As we see, considering twisted pair of above particles with "c" speed, we obtain valid approximation for photon energy. We should notice, that above hypothetical photon energy formula may be tested for pulsations close to Planck pulsation $\omega_P = 1/t_P$ treating Planck pulsation ω_P as the limit.

While we are considering in quantum mechanics a photon described by the commonly used energy formula $\hbar\omega$ we obtain known problems at Planck length scales that have vital meaning for quantum cosmology and for attempts to grand unification. [5]

Just introduced formula based on (97) does not crash at Planck time scales, because $\hbar\omega$ acts only as approximation for small energies.

4.1.3 Coulomb-like elementary charge interaction

We may describe elementary charge interaction as the result of formula:

$$\lim_{m \to m_P; r_s \to l_P/2\pi} V(r) \cdot (\gamma_r - 1) = \tag{99}$$

$$= m_P c^2 \left(\frac{1}{\sqrt{1 - \frac{l_P}{2\pi l_P}}} - 1 \right) \cdot \left(\frac{1}{\sqrt{1 - \frac{l_P}{2\pi r}}} - 1 \right) \approx \tag{100}$$

$$\approx E_P \cdot 4\pi\alpha \cdot \frac{l_P}{4\pi r} \approx \frac{\hbar c}{r} \cdot \alpha \tag{101}$$

where α is fine structure constant. This way we have obtained electrostatic potential for elementary charges expressed with natural units.

In this point we have to point out that limit considered here is in fact interaction between fields $V(r)\frac{V(r)}{mc^2}$, which can be interpreted as first approximation of interaction between two particles. This shows that in fact we may get in natural way fine structure constant approximation if we introduce interaction between two particles and consider it in first order approximation.

4.2 Covariant quantization - second approach

As it was shown in section 4.1 we can find quantum equations for gravity if we choose metric in proper way. In next step we construct Hamiltonian and quantize it in classical way. Still we can not always be sure that form of our metric give us Hamiltonian from which we could get quantum equations that will be easily interpreted by use

of the Copenhagen [6] interpretation of wave function. This remark lead us to second approach to quantization.

In this approach we have to notice that natural interpretation of wave function is easy for Cartesian and Minkowski metrics. For both of this spaces we have well defined time and space variable which are separated and do not have singularities which are artefacts of coordinate system. For that metrics wave function is well defined.

If we try to introduce quantization for other metric we start with problem of good definition of proper time, separation between space and time variables, artefact singularities. When we try to quantize metric of the form of equation (9) we have good definition of time, separation between space and time variables, but as we easily can see we have singularity for $r = r_s$ which is not the source of the field.

This problem is well known from at least 100 years and was solved by Eddington in 1924 [7]. He had proposed transformation to the isotropic coordinate using:

$$r = r_1\left(1+\frac{GM}{2c^2r_1}\right)^2 \tag{102}$$

$$r_1 = \frac{r}{2}-\frac{GM}{2c^2}+\sqrt{\frac{r}{4}\left(r-\frac{2GM}{c^2}\right)} \tag{103}$$

and metric takes the form:

$$ds^2 = \frac{(1-\frac{GM}{2c^2r_1})^2}{(1+\frac{GM}{2c^2r_1})^2}c^2dt^2-\left(1+\frac{GM}{2c^2r_1}\right)^4(dx^2+dy^2+dz^2) \tag{104}$$

As it can be shown, from this coordinate metric we have only one singularity in $r = 0$ which is physical and is source of the field.

We have to realize that metric which we choose in section 4.1 was first verification that can be rewritten as wave equation (see section 3.3.1). Of course it is not always the case for every metrics that we can take. Still we would like to have some generic method to quantize metrics that have easy physical interpretation.

Ansatz 1. *We can quantize gravity if the metric fulfill all the properties:*
- *time have proper local interpretation*
- $g_{ij} = 0$ *for all* $i \neq j$, *where* $i, j = 0, .., 3$
- *singularities are only point source of the field*

For that choose of the metric we can take Hamiltonian in covaraint form [8]

$$H = \frac{1}{2}g^{\alpha\beta}p_\alpha p_\beta \tag{105}$$

and quantize it to the form of

$$H = \frac{1}{2}g^{\alpha\beta}\widehat{p_\alpha}\widehat{p_\beta} \tag{106}$$

We choose to take representation of four-momentum in form of:

$$\widehat{p_\alpha} = (\hat{E},\hat{\mathbf{p}}) = (i\hbar\frac{\partial}{\partial t}, -i\hbar\frac{\partial}{\partial \mathbf{r}}) \tag{107}$$

This lead us to the commutation rule of form:

$$\{p^\mu, p_\nu\} = const.\delta^\mu_\nu$$

For that we can define equation of the form:

$$\frac{Dp^\xi}{dt} = \{H, p^\xi\} + \frac{\partial p^\xi}{\partial t} \tag{108}$$

That equation by definition we can rewrite to more simple form:

$$\frac{Dp^\xi}{dt} - \frac{\partial p^\xi}{\partial t} = \{H, p^\xi\}$$

and simplifying we get:

$$\Gamma^\mu_{\xi 0}p_\mu + \{H, p^\xi\}p^\xi = 0 \tag{109}$$

Which we can write in quantum form as:

$$\Gamma^\mu_{\xi 0}\widehat{p_\mu}\Psi + \{H, \widehat{p^\xi}\}\widehat{p^\xi}\Psi = 0 \tag{110}$$

This give us:

$$\Gamma^\mu_{\xi 0}\widehat{p_\mu}\Psi + c\widehat{p^\xi}\widehat{p^\xi}\Psi = 0 \tag{111}$$

For our metric we get two equations:

$$\Gamma^0_{10}\widehat{p_0}\Psi + c\widehat{p^1}^2\Psi = 0$$

$$\Gamma^1_{00}\widehat{p_1}\Psi + c\widehat{p^0}^2\Psi = 0$$

which we rewrite in the form of:

$$\Gamma^0_{10}g_{00}\widehat{p^0}\Psi + c\widehat{p^1}^2\Psi = 0$$

$$\Gamma^1_{00}g_{11}\widehat{p^1}\Psi + c\widehat{p^0}^2\Psi = 0$$

Here we see that $\Gamma^0_{10}g_{00} = -\Gamma^1_{00}g_{11}$ and if we add this two equations we get:

$$\widehat{p^1}^2\Psi + \widehat{p^0}^2\Psi = 0 \tag{112}$$

This is the plain wave equation. In this picture gravity can be seen as free wave function of particle with no mass.

In our opinion only external field or interaction between two massive particle can create in this picture effective mass of particle that have gravity mass in classical picture.

This should not surprise us. We consider our field locally and start quantize it in local neighborhood around singularity. In this case mass that we consider is in fact 0. In our definition of metric proper choose of mass is Komar Mass [9] which for vacuum solution of Einstein equation is always zero if we integrate on volume different than infinity.

This remark in natural way lead us to interpretation that our quantization is proper for this choose of metric. Our solution for quantum equation should be plain wave function with no mass for local frame.

5 Conclusions and open issues

In this article we try to shed new light on our understanding of gravity. We present that gravity can be seen as classic-like field that propagate in the vacuum as wave. This remark lead us to conclusion, that we may try to quantize that field and introduce effective theory of quantum gravity.

We may also conclude, that it is natural to consider gravity in (3+1) manifold. This choose give us opportunity to reintroduce time in quantum mechanic as regular dimension. In this way we can find natural interpretation for quantum equation that we have obtained.

We have also shown that foliation of spacetime is not unique for classical approach to gravity as field. We can get the same interpretation for classical field in two different choose of foliations. This problem disappear if we start to quantize our field. Depending on choose of (3+1) metric we get different quantum mechanic equation. We can interpret this as the choose of coordinate system.

In classical quantization we choose coordinate system for observer in infinity and for covariant quantization we choose local observer. We conclude, that we have to choose properly our metric depending on problem that we consider.

In section (4.1.2) we consider the case in which energy of our system is carried by photon. We see from equation (97) that energy of that photon will not change linear with its frequency for high energies close to Planck scales. This might be proposed as a test of our model.

In covariant quantization we remark that in local frame we see only massless particles traveling with speed of light. This picture will change if we introduce interaction with other particles for example other photon. It requires further investigation but we can propose test of our model for high frequency resonator where we can test interactions between photons that wave functions have Planck scales and interact between each other.

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Key words. General Relativity, Gravity, Quantum Gravity, Maxwell Equations

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